3D Matrix Transformations

Computer Graphics

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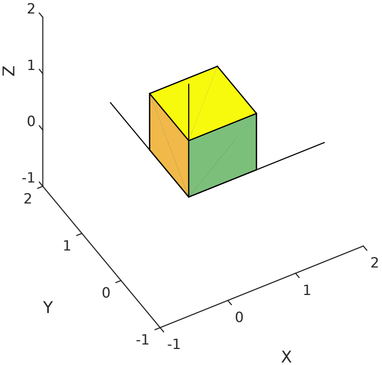
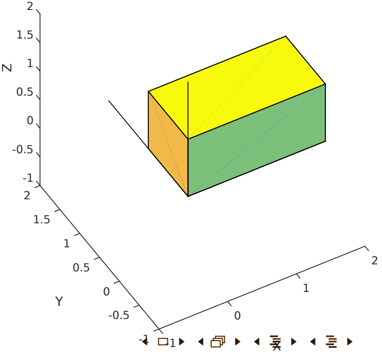
3D Matrix Multiplication

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{bmatrix}$$

3D Scale X

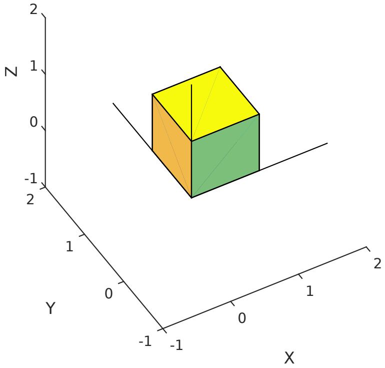
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x \\ y \\ z \end{bmatrix}$$



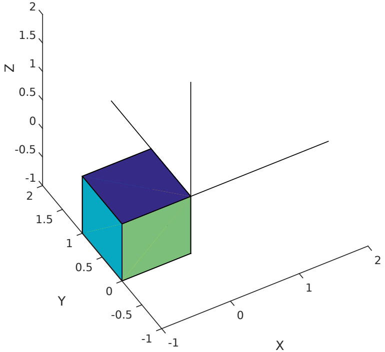


3D Flip XZ

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -x \\ y \\ -z \end{bmatrix}$$



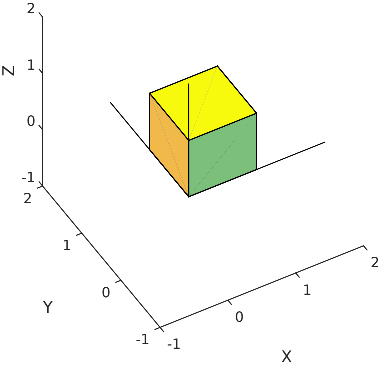
Before

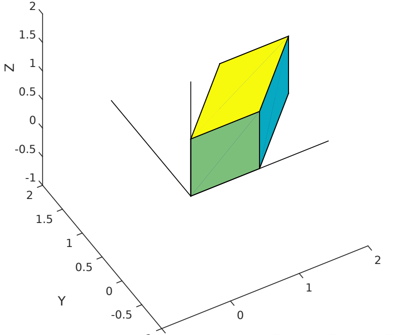


After

X Shear Along Y

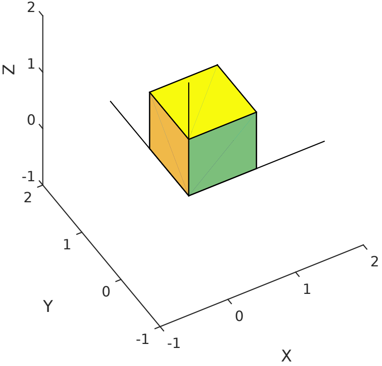
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + y \\ y \\ z \end{bmatrix}$$



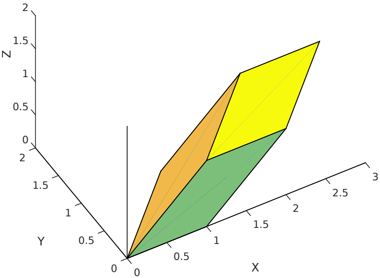


X Shear Along Y and Z

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x+y+z \\ y \\ z \end{bmatrix}$$



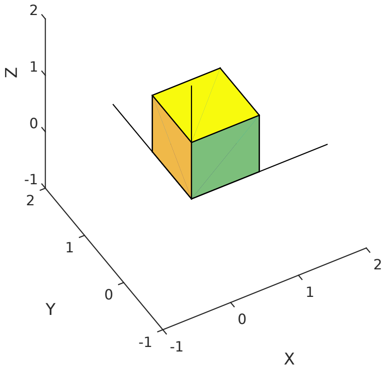
Before



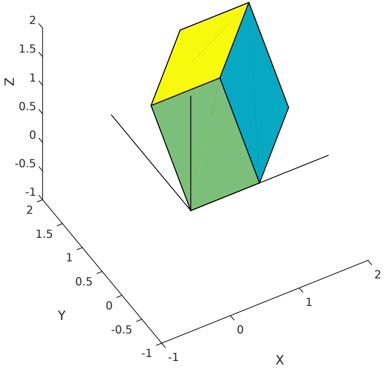
After

X Shear Along Y, Y Shear Along Z

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + y \\ y + z \\ z \end{bmatrix}$$



Before



After

X Shear Along Y, Y Shear Along Z

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + y \\ y + z \\ z \end{bmatrix}$$

Interactive Demo

3D Homogenous Coordinates

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & T_x \\ A_{21} & A_{22} & A_{23} & T_y \\ A_{31} & A_{32} & A_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Homogenous Coordinates

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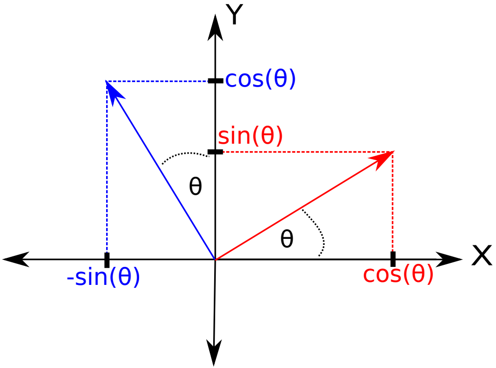
$$A \begin{bmatrix} x \\ 1 \end{bmatrix} = \begin{bmatrix} A^{3\times 3}x + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \\ 1 \end{bmatrix}$$

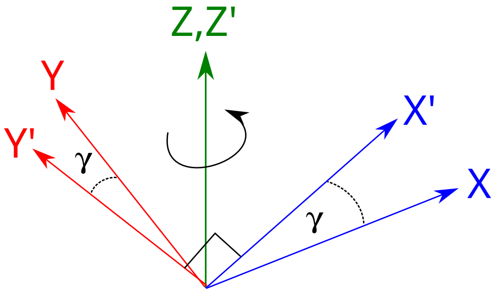
2D Rotation Matrix Design: Review



$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Rotation About Z (Roll)

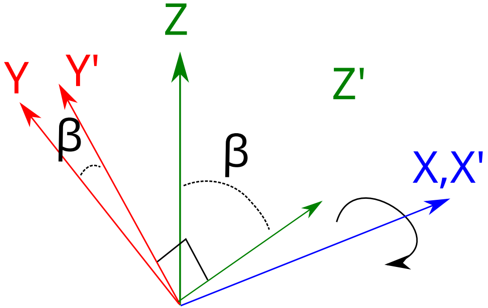
$$R_Z(\gamma) = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Just like the normal 2D *XY* rotation

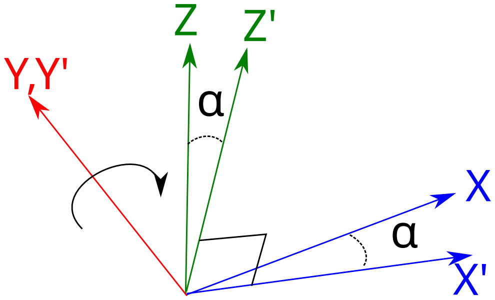
Rotation About X (Pitch)

$$R_X(\beta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\beta) & -\sin(\beta) \\ 0 & \sin(\beta) & \cos(\beta) \end{bmatrix}$$



Rotation About Y (Yaw)

$$R_Y(\alpha) = \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}$$



This one hurts the brain a little

Tait-Bryan Angles

Can chain these matrices together in any order, such as

$$R_{ZYX} = R_X(\beta)R_Y(\gamma)R_Z(\alpha)$$

$$R_{XYZ} = R_Z(\alpha)R_Y(\gamma)R_X(\beta)$$

Resulting matrix is always *orthogonal*

Furthermore, it is possible to reach any orientation with three angles in one of these configurations

Tait-Bryan Angles

Tait-Bryan Angles Demo